Mathematical Models in Marketing

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***Networks, Crowds, and Markets*, Chapter 16: information Cascades**

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based on <https://en.wikipedia.org/wiki/Information_cascade>

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**Key contribution**

Model of information cascades based on a simple Bayesian rule.

* It naturally accounts for power and fragility of the cascades.

**Information cascade** occurs when a person observes the actions of others and then – despite possible contradictions in his/her own private information signals – engages in the same acts. A cascade develops when people "abandon their own information in favor of inferences based on earlier people's actions".

**Example:** you come across 10 people looking at the sky in the middle of the street. You decide to look up too because you infer from others’ actions that something important might be happening up there.

*Note:* Here actions of others affect your actions indirectly – by changing your information. Alternatively, one may change his behavior also due to *direct benefit effects* – where conformity is directly rewarded – this, however, is distinct from information cascades.

**Requirements for a cascade to occur**

1. There is a decision to be made.
2. People make decisions sequentially.
3. People observe choices made by those who acted earlier.
4. Each person has *private information* hat helps guide his decisions.
5. People cannot observe private information that others know, but they can make inference about it from others’ actions.

**Experiment**

*States of the world*

1. Urn with 2 blues balls for every red ball (B)
2. Urn with 2 red balls for every blue ball (R)

*Prior probability* on each state:

One by one, participants draw a marble from this container. Possible draws are and for blue and red balls. Others do not see the choice, so draws are *private information*.

Participant makes a guess about whether the urn is R or B and announces the guess publically.

The first person guesses the urn of the color of the ball he has drawn. If the second student draws the same color ball as the first – then he announces the same color, otherwise, he is indifferent. We assume he announces the drawn color.

The cascade starts when two people announce two guesses of the same color (B,B) or (R,R). Such public evidence would dwarf any private information of the subsequent participant, and would trigger a chain reaction.

* Cascades can easily occur.
* Cascades can be wrong (1/3 \* 1/3 is the probability of a sequence of two wrong color announcements).
* Cascades can be fragile (imagine if two students cheated and showed their marbles two class – they could disrupt an existing cascade).

**General Cascade Model**

*States of the world*

1. G: the option is a good idea
2. B: the option is a bad idea

*Prior probability* on each state:

*Payoffs*

*Accept*

*Good:*

*Bad:*

*Reject*

*Prior without any information, we have*

*Private Information (signals)*

A person's signal telling them to accept is denoted as *H* (a high signal, where high signifies he should accept), and a signal telling them not to accept is *L* (a low signal).

The model assumes that when the correct decision is to accept (with prior p), individuals will be more likely to see an *H*, and conversely, when the correct decision is to reject, individuals are more likely to see an *L* signal.

This is the probability of *H* when the correct action is to accept, or . Similarly, is the probability that an agent gets an *L* signal when the correct action is reject. If these likelihoods are represented by *q*, then *q* > 0.5. This is summarized in the table below.

|  |  |  |
| --- | --- | --- |
| **Agent signal** | **True probability state** | |
| **Reject** | **Accept** |
| *L* | *q* | 1-*q* |
| *H* | 1-*q* | *q* |

*Analysis*

**The first agent** determines whether or not to accept solely based on his own signal. This decision can be explained using Bayes rule

{\displaystyle {\begin{aligned}P\left(A|H\right)&={\frac {P\left(A\right)P\left(H|A\right)}{P\left(H\right)}}\\&={\frac {P\left(A\right)P\left(H|A\right)}{P\left(A\right)P\left(H|A\right)+P\left(R\right)P\left(H|R\right)}}\\&={\frac {pq}{pq+\left(1-p\right)\left(1-q\right)}}\\&>p\end{aligned}}}

If the agent receives an *H* signal, then the likelihood of accepting is obtained by calculating The equation says that, by virtue of the fact that *q* > 0.5, the first agent, acting only on his private signal, will always increase his estimate of *p* with an *H* signal. Similarly, it can be shown that an agent will always decrease his expectation of *p* when he receives a low signal. If the value, *V*, of accepting is equal to the value of rejecting, then an agent will accept if he believes *p* > 0.5, and reject otherwise. Because this agent started out with the assumption that both accepting and rejecting are equally viable options (*p* = 0.5), the observation of an *H* signal will allow him to conclude that accepting is the rational choice.

**Lemma for Multiple Signals** In general, the *n*th agent considers the decisions of the previous *n*-1 agents, and his own signal. He makes a decision based on Bayesian reasoning to determine the most rational choice.

Where *a* is the number of accepts in the previous set plus the agent's own signal, and *b* is the number of rejects. Thus, .

*Majority voting over signals*

**The nth agent from sequential decision-making**

Assumptions: previous decisions are available while the nth decision maker cannot know the previous private information from the previous n-1 agents.

As long as the difference between the number of accepting and rejecting is more than 2, a cascade begins.

The likelihood of information cascade given large group is very high.

Managerial implications

Organizational Behavior: Group Thinking, Brainstorm before Consensus

Marketing: New Product Promotion on Social Media (Adoption information)

**Discussion**

1. Making private information public can prevent or break information cascade.
2. When extending the model with binary variable to non-binary variables, the general intuition holds.
3. When giving different weights for public versus private information, the general intuition holds.
4. Payoffs vary from person to person. The model assumes that when the correct decision is to accept (with prior p), individuals will be more likely to see an *H*, and conversely, when the correct decision is to reject, individuals are more likely to see an *L* signal. It is possible that this matrix is not symmetric but the general intuition holds.
5. Not all previous information are available.
6. There is no contextual or psychological factor included in this model.
7. McCoy’s Paper on Nature.
   1. Both of them use Bayes’ Rule and talk about decision-making.
   2. What if every agent can report his confidence level?
   3. Can McCoy’s rule extend to group thinking?

**Take-Home Message**

When people can see what others do but not what they know, there is an initial period when people rely on their own private information, but as time goes on, the population can tip into a situation where people begin ignoring their own information and following the crowd. But, they are behaving fully rationally.